LECTURES OF MATHEMATICS

2018-2019

Al- Maaʾrif   College University

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Topics :

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Grading Course :

Quizzes	**5%**
Reporting	**5%**
Course testing	**30%**
Final	**60%**

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Reference :

1-A.G.Purcell,,calculus and geometry analytic,, translation ,By, Dr Ali Azizه,Univ, of ,Al-Mosul ,second edition ,2005

2-Barnard, (introduction in a linear algebra),translation by, Dr . Adel ghassan ,univ ,of Baghdad 1990 .

Coues outline :

The textbook covers the subject in detail ,so I hope students to prepare for each lecture by reading the assigned section in advance . Below is the tentative detailed outline of syllabus for the entire quarter

[1.1] Integration and its application .

Some laws of integration .

$\int u^n\, du = \frac{u^{n+1}}{n+1} + c$ for $n \neq -1$.

$= \text{Ln}\, u + c$ for $n = -1$

1) $\int \sin u\, du = -\cos u + c$ 2) $\int \cos u\, du = \sin u + c$

3) $\int \sec^2 u\, du = \tan u + c$. 4) $\int \csc^2 u\, du = -\cot u + c.$

5) $\int \sec u \tan u\, u\, du = \sec u + c$. 6) $\int \csc u\, \text{cotan}\, u\, u\, du = -\csc u + c$

7) $\int \tan u\, du = \int \frac{\sin u}{\cos u}\, du . = -\int \frac{-\sin u}{\cos u}\, du = -\ln \cos u + c$.

8) $\int \cot u\, du = \ln \sin u + c$. 9) $\int \sec u\, du = \ln(\sec u + \tan u) + c$.

10) $\int \csc u\, du = \ln(\csc u - \text{cota n}\, u) + c$.

11) $\int e^u\, du = e^u + c$.

Example. $\int x^3\, dx \Rightarrow \int x^3\, dx = \frac{x^{3+1}}{3+1} = \frac{x^4}{4} + c$

[1. 2] Some formula of integration.

If f (x) , g (x) are differentiable function t hen .

1) $\int f(x)dx = f(x) \Rightarrow \int dx = x + c.$ 2) $\int c\, f(x)dx = f(x) = c \int f(x) + c.$

3) $\int (f(x) + g(x))dx = \int (f(x)dx + \int g(x))dx + c.$

Example . Find the resulting of the integration.

1) $\int (x^2 - 3x + 5)\, dx = \int x^2 dx - 3\int x\, dx + 5\int dx$

$$= \frac{x^3}{3} - 3\frac{3x^2}{2} + 5x + c .$$

2) $\int (x^2 + 3x^2)^2\, dx$; 3) $\int (\frac{x^2 + 3x^2 - 2}{x^2})\, dx$.

[1 . 3] Definite integration .

First we show that the definite integration for the continuous function f(X) . from a into b , which is denoted by $\int_a^b f(X)$. That means in Geometry the enclosed area between the curve y =f (X) and x- axis and the line x = a or x =b . Denoted A for the area and curve y = f(X) , and x- axis and the vertex line $P_1 P_2$, the variable capitates line . P $_3$P

[1 . 4] Definition. If the function is continuous on the interval [a , b] ,and inverse derivative of f(x) in [a, b] then .

$\int_a^b f(x)\, dx = f(b) - f(a)$.

Example. Calculate $\int_2^3 x^3\, dx$

Solution. $\int_2^3 x^3\ dx = [\ \frac{x^4}{4}\ ]_2^3 = \frac{(3)^4}{4} - \frac{(2)^4}{4} = \frac{81}{4} - \frac{16}{4} = \frac{65}{4}$

[1 . 5] Multiplication integral (double integral) .

a) Binary integral : Consider that F(x , y) a function is continuous in the area R. then

$$\int_A F(x, y)\ dA = \lim_{\Delta A \to 0} \sum_{k=1}^{n} f(x_k\ ,\ y_k)A_k$$

The laws to compute (account) binary are the integrals.

We can finding the dual integrals

$$\int_R \int F(x, y)\ dX\ dy$$

a) $\int_R \int F(x, y)\ dXdy = \int_{y_1}^{y_2} \int_{h_1(y_1)}^{h_2(y_2)} F(x, y)\ dX\,dy = \int_{y_1}^{y_2} [\int_{h_1(y_1)}^{h_2(y_2)} F(x, y)\ dX\ ]\ dy$

We show that $y_1 < y < y_2$, $h_1(y) < x < h_2(y)$

a-for account the integral. $\int_R \int F(x, y)\ dy\ dx = \int_{x_1}^{x_2} \int_{g_1(x_1)}^{g_2(x_2)} F(x, y)\ dy\ dx$

$= \int_{x_1}^{x_2} [\int_{g_1(x_1)}^{g_2(x_2)} F(x, y)\ dX\ ]\ dy.$

Example. Find the output of the following.

1)$\int_x^1$ (x , y) dy dx (2) $\int_0^2 \int_{2x}^{x^2} (x - 2y)\ dy\ dx$ (3) $\int_0^1 \int_1^3 (y^2 + 3xy^2)\ dx\ dy$

Solution: 1) $\int_0^2 \int_{2x}^{x^2} (x - 2y)\ dy\ dx = \int_0^2 [\int_{2x}^{x^2} (x - 2y)\ dy\ ]\ dx$

$= \int_0^2 [\int_{2x}^{x^2} x\,dy - \int_{2x}^{x^2} 2y\ dy\ ]\ dx = \int_0^2 [\ xy - y^2\ ]_{2x}^{x^2}\ dx = \int_0^2 (\ x(x^2) - (x^2)^2 - (2x(x)) - (2x^2)\ )\ dx$

$= \int_0^2 (x^3 - x^4 - 2x^2 + 4x^2)\ dx = \int_0^2 (-x^4 + x^3 + 2x^2)\ dx = [\ \frac{-x^5}{5} + \frac{x^4}{4} + \frac{2x^3}{3}]_0^2$

$= [\ \frac{-32}{5} + \frac{16}{4} + \frac{16}{3}] = [\ \frac{-32}{5} + \frac{48+64}{12}] = [\ \frac{-32}{5} + \frac{112}{12}] = [\frac{-384+560}{60}] = \frac{-176}{60} = -2.9$

[1 . 6] The triple integral .

By the same style. We account the binary integration also we account the triple integral.

$$\int_1^3 \int_{-1}^1 \int_1^2 3(x^2y + y^2z)\ dx\ dy\ dz.$$

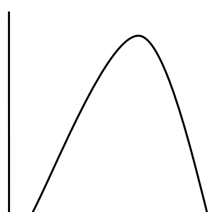
Application of Multiple integration (binary, triple) .

Bounded (enclosed) area between two curves. To find area A is bounded by two curve $y_1 = f(x)$, $y_2 = f(x)$, from above or down and two lines x= a, x = b

$A = \int_a^b f_2(x)\ dx - \int_a^b f_1(x)\ dx = \int_a^b [f_2(x) - f_1(x)]\ dx$

$= \int_a^b \int_{f_1(x)}^{f_2(x)} dy\ dx$

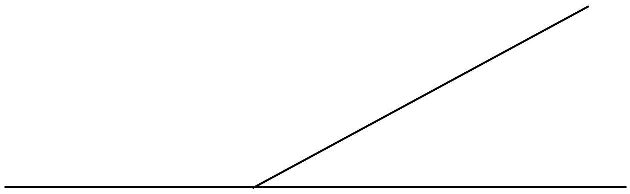
Example. By using the binary integration, find the enclosed area between $y = x$, $y = 4x - x^2$



Solution . y= $f_2(x)$

X = $4x - x^2$ y =$f_1(x)$

$x^2 - 4x + x = 0 \implies x^2 - 3x = 0$



$\implies x(x-3) = 0 \quad 0 \qquad 3$

$\implies x = 0$ or $x = 3$

$\implies a = 0$ or $b = 3$

$f(1) = 1$, $f(1) = 4 - 1 = 3 \implies f_2(x) = 4x - x^2$

$f_1(x) = x$

$\therefore A = \int_a^b \int_{f_1(x)}^{f_2(x)} dy\,dx \quad cA = \int_0^3 \int_{x)}^{4x-x2)} dy\,dx = \int_0^3 [y]_x^{4x-x} dx$

$= \int_0^3 4x - x^2 - x)\ dx = \int_0^3 3x - x^2\ dx = [\frac{3x^2}{2} - \frac{x^3}{3}]_0^3 = \frac{27}{2} - \frac{27}{3} = \frac{27}{2} - 9 = \frac{9}{2}$

Some integral Methods

1) Integration by parts.

$d(u.v) = u.\,dv + v\,du \implies d(u.v) = v.\,du + u\,dv$ take the integration to both side

$\int u.dv = \int u.dv - \int v.du \implies \int u.dv = u.v - \int v.du$

Example . Evaluate the following integration $\int x\,e^x\,dx$

Solution : Let u = x , dv = $e^x dx$

du = dx , v = e^x

$\int u.dv = u.v - \int v.du = \int x\,e^x\,dx$

U		dv
X		e^x
1	+	$-e^x$
0	-	$-e^x$
		e^x

Example: $\int x\,lin\,x\ dx$, $\int sinx\,e^x\,dx$, $\int x\cos 2x\ dx$ (. WH)

2) integration Rational Function by partial Fraction .

A) Linear Factors .

For each factor of the form $(ax+b)^m$, the partial fraction contain the following sum of m partial fraction. $\frac{A_1}{(ax+b)1} + \frac{A_2}{(ax+b)2} + \frac{A_3}{(ax+b)3} + \ldots + \frac{A_m}{(ax+b)m}$ where $A_1, A_2, A_3, \ldots, A_m$ are constant to be determine

Example : evaluate the following integrals , $\int \frac{dx}{x^2+x-2}$

Solution: $\int \frac{dx}{x^2+x-2} = \frac{1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$

$\frac{1}{(x+2)(x-1)} = \frac{A(x-1)+B(x+2)}{(x+2)(x-1)}$

1 = A(x – 1) + B(x - 1)

1 = Ax – A + Bx + 2B

$0.x =$ (A+ B) x + (2B – A)

$A + B = 0 \implies$ B = -A

$2B - A = 1 \implies$ B = $\frac{1}{3}$ hence A = - $\frac{1}{3}$

$\frac{1}{(x+2)(x-1)} = \frac{-\frac{1}{3}}{(x+2)} + \frac{\frac{1}{3}}{(x-1)}$

$\int \frac{dx}{x^2+x-2} = \int \frac{-\frac{1}{3}}{(x+2)} + \frac{\frac{1}{3}}{(x-1)} dx = -\frac{1}{3} \int \frac{dx}{(x+2)} + \frac{1}{3} \int \frac{\frac{1}{3}}{(x-1)}$

$= \frac{1}{3}$ lin | x - 1| - $\frac{1}{3}$ lin |x+2| + c $\rightarrow \int \frac{dx}{x^2+x-2} = \frac{1}{3}$ lin| $\frac{(x-1)}{(x+2)}$ + c

2)) $\int \frac{2x+4}{x^2-2x^2}$ dx (H.W)

3) Quadratic factors :

For each factor of the form (ax +bx + c)^m , the partial fraction contain the following sum of m partial fraction. $\frac{A_1x+B_1}{(ax^2+bx+c)1} + \frac{A_2x+B_2}{(ax^2+bx+c)2} + \ldots + \frac{A_mx+B_m}{(ax^2+bx+c)m}$ where $A_1, A_2, A_3, \ldots, A_m, B_1, B_2, \ldots, B_m$ are constant to be determine

Example : evaluate the following integrals , $\int \frac{x^2+x-2}{3x^3-x^2+3x-1}$

$\int \frac{x^2+x-2}{3x^3-x^2+3x-1} dx = \int \frac{x^2+x-2}{(3x-1)(x^2+1)} dx \rightarrow \frac{x^2+x-2}{(3x-1)(x^2+1)} = \frac{A}{(3x-1)} + \frac{Bx+c}{(x^2+1)} = \frac{A(x^2+1)+(Bx+c)(3x-1)}{(3x-1)}$

$x^2 + x - 2$ = AX^2 + A + 3B X^2 – BX + 3CX – C

$x^2 + x - 2$ = (A + 3B)X^2 + (3C –B)X+ (A –C)

$A + 3B = 1$ . . . . ……….(1)

3C – B = 1 . . . . ………….(2)

C = -2 $\implies$ A = C-2………(3)

Hence A = $\frac{-7}{5}$ $, \implies B = \frac{4}{5}, \implies C = \frac{3}{5}$

$\int \frac{x^2+x-2}{3x^3-x^2+3x-1} = \frac{-7}{5} \int \frac{dx}{(3x-1))} + \frac{4}{5} \int \frac{dx}{(x^2+1)} + \frac{3}{5} \int \frac{dx}{(x^2+1)}$

$= \frac{-7}{5} lin \mid 3x - 1 \mid + \frac{4}{5} lin \mid (x^2 + 1 \mid + \frac{3}{5} lin \mid (x^2 + 1 \mid$ + c

Notes :

If the degree of numerator grater than or equal the degree of denominator , first divided the denominator in to numerator to get the polynomial plus proper fraction .

Example : L evaluate $\int \frac{2x^2 - 4x^2 - x - 3}{x^2 - 2x - 3}$

Solution : $\frac{2x^2 - 4x^2 - x - 3}{x^2 - 2x - 3} = 2x + \frac{5x - 3}{x^2 - 2x - 3}$

$\int \frac{2x^2 - 4x^2 - x - 3}{x^2 - 2x - 3} = \int 2x\,dx + \int \frac{5x-3}{x^2 - 2x - 3}$ dx

$\int \frac{5x-3}{x^2 - 2x - 3}$ dx $= \frac{5x-3}{(x+1)(x-3)} = \frac{A}{C(x+1)} + \frac{B}{(x-3)}$……………………….(1)

$\frac{5x-3}{(x+1)(x-3)} = \frac{A(x-3)+B(x+1)}{(x-3)}$

$5X - 3$= AX – 3A+ BX + B

$5X - 3$ = AX + BX – 3A + B

$5X - 3 = (A + B)X - 3A + B$

$A + B = 5$ → B = 5 –A ………..(1)

$-3A + B = -3$ ………………………..(2)

$-3A + 5 - A = -3$ → $-4A = 8$ → A =2, B= 3

$\frac{5x-3}{(x+1)(x-3)} = \frac{2}{(x+1)} + \frac{3}{(x-3)}$ ……………………….(2) substitute (2(into(1) we obtan

$\int \frac{5x-3}{x^2 - 2x - 3}$ dx = $\int 2x\,dx + \int \frac{2}{(x+1)}$ dx + $\int \frac{3}{(x-3)}$ dx = X^2 + 2 lin (x+ 1) 3 lin (x- 3) + c

example : $\int_{-1}^{0} \frac{x\,(x2\ -1)}{(x2\ +1)(x-1)}$ dx

solution : $\int_{-1}^{0} \frac{x\,(x2\ -1)}{(x2\ +1)(x-1)}$ dx= $\int_{-1}^{0} dx + \int_{-1}^{0} \frac{(x\ -1)}{(x2\ +1)}$ dx == x] $^{+} \int_{-1}^{0} \frac{(x)}{(x2\ +1)} dx + \int_{-1}^{0} \frac{dx}{(x2\ +1)}$

= 0 + 1 + $\frac{1}{2}$ lin (x + 1)] – $\tan^{-1}$ x]

Evaluate the following : $\int_{0}^{1} \frac{dx}{(x2\ +1)(x+1)}$, $\int \frac{x^3 - x}{(x2\ +1)(x-1)}$

[2 .1] Chapter two : Differentiation equation [D. E . Q]

A) Degree of equation . Representation the power its upper differentiation

$(y'')^{2} + (y')^{4}$ = cos y …………… (2 - degree)

B) Order of equation . representation upper of differentiation appear in it the equation

$(y')^{4} + (y'')$ = x e^x ………………….. (from 1 - ord)

Example: 1) $(y')^{4} + (y'')^{3} + (y''')^{2}$ = sin x 3 rd - order – 2nd degree

2)-$\frac{d^2y}{dx^2}$ + x^2 + $\frac{d^3y}{dx^3}$ + sin (xy) = e^x …from 3 rd - order , 1 nd -degree

3 (y') + 2 (y'') + y^2 = 2x

4-Xy + $2(y')^2$ = tan x

5- $e^x + \frac{dy}{dx}$ + x^3 $(\frac{d^2y}{dx^2})^2$ = 5x

[2.2] : Type of [D . E . Q] and methods of solve it.

We are know there are tow Differentiation equation **are** Ordinary and partial . Hence to solve these . from first degree , first order we have to methods are Variable separable and linear equation

[1] Variable separable method D .E (1st- order)

Generally from 1st order D . Eq if the D . Eq. can be written in the from p(x) dx + Q (y) dy = C

Example : $y' = \frac{dy}{dx}$ = y

solution

dy = y dx divided by y both sides

$\frac{dy}{y}$ = dx integrate by y both sides

ln y = x + c take e^x both sides

ln y = $e^{x + c}$

Y = c e^x G.S

Example: $\frac{dy}{dx}$ = y' = $e^{x + y}$ (H . W)

[2] Linear equation method of 1st

The D. E .Q of the form $\frac{dy}{dx}$ + p(x) y = Q (x) ……….(1)

Are called "linear 1.st order D.E.Q " . when P and Q are function of x or Constance . to solve this Eq . we multiplication both sides by an integral factor denoted (I. F) , which always

I. F = $e^{\int p(x)dx}$

Steps solution

1- Find I. F
2- Take following law y. I.F = $\int (I . F) Q(x) dx$
3- Integral both sides .

Example : Find the general solution of the following equation .

$\frac{dy}{dx} + \frac{dy}{x} = \sin x$

Solution First determine 1) $p(x) = \frac{1}{x}$,

2) $Q(x) = \sin x$

3- Find (I. F) $= e^{\int p(x)dx} = e^{\ln x\, dx} = x$

y. I.F $= \int (I.F)Q\,x\,dx \Rightarrow y \cdot x = \int (x \cdot \sin x\, dx)$

u	dx
x	+sin x
1	- coos x
0	- sin x

$y \cdot x = -x \cos x + \sin x + c$

$y = -\cos x + \frac{\sin x}{x} + \frac{c}{x}$ divided by x both sides

Example : find the solution for equation

$\frac{dy}{dt} + 3y = e^t$ when $y(0) = 1$

Solution

$P(t) = 3\,dt$, $Q(t) = e^{3t}$

I. F $= e^{\int p(t)dt} = e^{\int 3dt} = e^{3t}$

$y \cdot (I.F) = \int (I.F)Q(x)\,dx = \int e^{3t} \cdot e^t\, dt$

$y \cdot e^{3t} = \int e^{4t} \cdot dt = 4e^{4t} + c$ divided by e^{3t} both sides

$y = 4e^t + c\frac{1}{e^{3t}}$ G . S

Hence $1 = 4e^0 + c\frac{1}{e0}$

$1 = 4 + c \Rightarrow C = -3$

$Y = 4e^t - \frac{3}{e^{3t}}$ par .s

Example : A-Solve by using linear method.

1) $\frac{dy}{dx} + 5y = 50$; 2) $\frac{dy}{dx} + \frac{3y}{x} = \sin x$; 3 $-y' + x e^y = e^y$

4- $\frac{dy}{dx} + 2xZ = 2x$ when $z(0) = 1$

B -Using variable method for solve the following

1-) $\frac{dy}{dx} = xy^2 - y^2$ where y (0)= 2 ; 2) ($\frac{d\ y}{y^2}$) = (x - 1) dx , 3) 4xy dx + (x^2 + 1) dy

[2. 3] The exact differential equation .

General formal to the differential equation M (x,y) dx + N (x,y) dy =0 be exacts if the following $\frac{\partial m\ (x,y)}{\partial y} = \frac{\partial n\ (x,y)}{\partial x}$ is hold.

Method to solve the exact differential equation .

A) we derive M(x ,y) for y and remains x is constant .

B) we derive N(x, y) for x and remains y is constant.

To make sure if $\frac{\partial m\ (x,y)}{\partial y} = \frac{\partial n\ (x,y)}{\partial x}$

C) we Integrate M(x ,y) for x and make optional constant to y such that $\emptyset\ (y)$ from this, we get the equation number (1).

D) derive the equation number (1) in (c). then equal it with N (x,y). resulting from above we will get the derivative of $\emptyset\ (y)$..

E) integrate the $\emptyset\ (y)$ and after that compensation result of integration with equation number one to get the special solution .

Example : $(3x^2 + 3xy^2)$ dx+ $(3x^2 y - 3y^2 + 2y)$dy =0

Solution:

M(x,y) = $(3x^2 + 3xy^2)$ dx and N (x,y)= $(3x^2 y - 3y^2 + 2y)$dy

$\frac{\partial m\ (x,y)}{\partial y} = 6xy$

$\frac{\partial n\ (x,y)}{\partial x} = 6xy$

Hence $\frac{\partial m\ (x,y)}{\partial y} = \frac{\partial n\ (x,y)}{\partial x}$ then the equation is exact

F(x,y) = $\int m(x,y)dx = \int (3x2 + 3xy2)\ dx$

$= x + \frac{3}{2}\ x\ y + \emptyset\ (y).$ .... (1)

$\frac{\partial}{\partial y}(x + \frac{3}{2}\ x\ y + \emptyset\ (y).)$

$3x\ y + \emptyset\ (y).) = 3x^2 y - 3y^2 + 2y$

$\emptyset\ (y).) = - 3y^2 + 2y)$

$\int (\emptyset\ (y). = \int (-\ 3y2 + 2y) = y^3 + y^2 + c$

Fanilly $X + \frac{3}{2}\ X^2 y^2 - y^3 + y^2 + C$.

Chapter three : (Matrix)

[3 .1]Definition: Size of matrix : the dimension (size) of A is (m × n) read (m by n) , * if m=n we say that A is a square matrix , of order n . * if $a_{ij} = 0$ for $i \neq j$ we say that A is diagonal matrix .

* the elements a_{11} , a_{22} , a_{33}, …., a_{nn} form the main diagonal of A

[3 . 2] Definition 8: Matrix multiplication: the product of A and B is the matrix C = [c_{ij}] , such that , $C_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \dots + a_{ip} b_{pj}$; $A_{m\times p} \cdot B_{p \times n} = C_{m \times n}$,

$$\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}} \underset{3\times 4}{\begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}} = \underset{2\times 4}{\begin{bmatrix} & & & \\ & & 26 & \end{bmatrix}}$$

$c_{23} = a_{21}xb_{13}+a_{22}xb_{23}+a_{23}xb_{33}$

(2x4)+(6x3)+(0x5)=26

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} & & & \overset{c_{14}}{13} \\ & & 26 & \end{bmatrix}$$

(1x3)+(2x1)+(4x2)=13

Exercise :let $A=\begin{bmatrix} 2 & 3 & -4 \\ 1 & 2 & 3 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 1 \\ -2 & 2 \\ 5 & -3 \end{bmatrix}$

$$C = A \times B = \begin{bmatrix} 2(3)+3(-2)+(-4)(5) & 2(1)+3(2)+(-4)(-3) \\ 1(3)+2(-2)+3(5) & 1(1)+2(2)+3(-3) \end{bmatrix} = \begin{bmatrix} -20 & 20 \\ 14 & -4 \end{bmatrix}$$

Rules of Matrix Arithmetic

$A=\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$ $B=\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

Multiply A·B: Multiply B·A

$A \cdot B=\begin{bmatrix} -1 & -2 \\ 11 & 4 \end{bmatrix}$ $B \cdot A=\begin{bmatrix} 3 & 6 \\ -3 & 0 \end{bmatrix}$

Therefore: $A \cdot B \neq B \cdot A$

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Therefore: $A \cdot B \neq B \cdot A$

Determinant of a 2x2 matrix

If A is a 2x2 matrix

A= $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Then we define

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \det(A) = a_{11}a_{22} - a_{12}a_{21}$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Example: if A= $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ then det A = |A|= 3-2=1

If A is a 3x3 matrix $A=\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Then we define

$$\det(A)=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Example 2

$$\det(A)=\begin{vmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{vmatrix} = 1\begin{vmatrix} 0 & 2 \\ -1 & 2 \end{vmatrix} -5\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} -3\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix}$$

$$=1[0*2 - (-1*2)] - 5\,[1*2-2*3] - 3[-1*1-0*3]=25$$

If $A=\begin{pmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{pmatrix}_{3\times 2}$ then there are two methods to compute the det A

Method 1: det A=a1 det $\begin{pmatrix} b2 & c2 \\ b3 & c3 \end{pmatrix}$ - b1 det $\begin{pmatrix} b1 & c1 \\ b3 & c3 \end{pmatrix}$ + c1 det $\begin{pmatrix} b2 & c1 \\ b2 & c2 \end{pmatrix}$

Exercise: A= $\begin{pmatrix} 2 & -1 & 5 \\ 5 & 2 & 3 \\ 1 & 2 & 2 \end{pmatrix}$

det A = 2det $\begin{pmatrix} 2 & 3 \\ 2 & 2 \end{pmatrix}$ - (-1) det $\begin{pmatrix} 5 & 3 \\ 1 & 2 \end{pmatrix}$ + 5det $\begin{pmatrix} 5 & 2 \\ 1 & 2 \end{pmatrix}$ = 2 (4 - 6) + 1(10-3) + 5 (10-2) = -4 +7+40=43

Exercise : A = $\begin{pmatrix} 1 & 2 & 3 & 0 \\ 3 & 1 & 4 & 2 \\ 5 & 2 & 1 & 10 \\ 2 & 3 & 1 & 0 \end{pmatrix}$ then det A= 1det $\begin{pmatrix} 1 & 4 & 2 \\ 2 & 1 & 10 \\ 3 & 1 & 0 \end{pmatrix}$ – 2det $\begin{pmatrix} 3 & 4 & 2 \\ 5 & 1 & 10 \\ 2 & 1 & 0 \end{pmatrix}$

$$+\,3\det\begin{pmatrix}3 & 1 & 2\\ 5 & 2 & 10\\ 2 & 3 & 0\end{pmatrix} - 0\det\begin{pmatrix}3 & 1 & 4\\ 5 & 2 & 1\\ 2 & 3 & 1\end{pmatrix}$$

Complete the solution?

Method(2)

Duplicate Column Method – for 3x3

From the previous slide:

$$\det(A)=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix} = a_{11}\begin{vmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{vmatrix} - a_{12}\begin{vmatrix}a_{21} & a_{23}\\ a_{31} & a_{33}\end{vmatrix} + a_{13}\begin{vmatrix}a_{21} & a_{22}\\ a_{31} & a_{32}\end{vmatrix}$$

$$= \det(A)= a_{11}a_{22}a_{33}+ a_{12}a_{23}a_{31}+ a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

$$\left|\begin{matrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{matrix}\right|\begin{matrix}a_{11} & a_{12}\\ a_{21} & a_{22}\\ a_{31} & a_{32}\end{matrix}$$

Example 3: $\det(A)=\left|\begin{matrix}1 & 5 & -3\\ 1 & 0 & 2\\ 3 & -1 & 2\end{matrix}\right|\begin{matrix}1 & 5\\ 1 & 0\\ 3 & -1\end{matrix}$ $=0+30+3-0-(-2)-10=25$

[3.3]The Invers of matrix

The scientific method include to find A^{-1} establish the Matrix [A : I_n] from sample n x n and conduct first row operations turn it into [I_n : A^{-1}] . Notice, all thing we conduct on row from A , should conduct it also on opposite row from I_n .

Method to find the invers of matrix

we take the Matrix [A : In] and turn into the format of the reduced row format [C : D] , when the matrix C be from type n x n it is from format row equal to the format A . let D1 ,D2 , . . ,Dn is the column n to matrix D . hence give us the matrix [C : D] for the following linear system CX=D. Now we have one probability is C= I_n and I_n x =x = D so we have obtained on A^{-1}

Example : finde the invers of the matrix

$$A=\begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 3\\ 5 & 5 & 1\end{bmatrix}$$

Solution:To find A^{-1} we make the following

$$\begin{array}{c} \\ [A : In] = \end{array}\begin{array}{ccc|ccc} & A & & & I_3 & \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 5 & 5 & 1 & 0 & 0 & 1 \end{array}\begin{array}{c} \\ r_1 \\ r_2 \\ r_3 \end{array} \qquad r_1\ (5) + r_3$$

$$\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & -4 & -5 & 0 & 1 \end{array}\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \qquad \text{divid } r_2 \text{ on 2 we get}$$

$$\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & -4 & -5 & 0 & 1 \end{array} \qquad \text{subtraction } r_2 \text{ from } r_1 \text{ to get}$$

$$\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & 1 & \frac{-1}{2} & 0 \\ 0 & 1 & \frac{3}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & -4 & -5 & 0 & 1 \end{array} \qquad \text{multiplication } r_3 \text{ in } \frac{-1}{4} \text{ to get}$$

$$\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & 1 & \frac{-1}{2} & 0 \\ 0 & 1 & \frac{3}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{5}{4} & 0 & \frac{-1}{4} \end{array} \qquad \text{multiplication } r_3 \left(\frac{-3}{2}\right) + r_2 \text{ to get}$$

$$\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & 1 & \frac{-1}{2} & 0 \\ 0 & 1 & 0 & \frac{-15}{8} & \frac{1}{2} & \frac{3}{8} \\ 0 & 0 & 1 & \frac{5}{4} & 0 & \frac{-1}{4} \end{array} \qquad \text{multiplication } r_3 \left(\frac{1}{2}\right) + r_1 \text{ to g}$$

$$D = \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{13}{8} & \frac{-1}{2} & \frac{-1}{8} \\ 0 & 1 & 0 & \frac{-15}{8} & \frac{1}{2} & \frac{3}{8} \\ 0 & 0 & 1 & \frac{5}{4} & 0 & \frac{-1}{4} \end{array} = C$$

Hence $C = I_3$ we conclude that $D = A^{-1}$

Hence $A^{-1} = \begin{bmatrix} \frac{13}{8} & \frac{-1}{2} & \frac{-1}{8} \\ \frac{-15}{8} & \frac{1}{2} & \frac{3}{8} \\ \frac{5}{4} & 0 & \frac{-1}{4} \end{bmatrix}$ To make sur that your solution is true should be

Corollary (1): if $|A| \neq 0$, then

$A\,A^{-1} = I_n$ also $|A\,A^{-1}| = |A||A^{-1}| = |I_n| = 1$ (H W)

Example : let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -1 \\ 2 & -1 \end{bmatrix}$, finde the invers of the matrix (H.W)

[3.4] Adjoint of Matrix

Let A= [an] a matrix from type (nxn) . the matrix is called adjA from type (nxn) which it elements i,j is cofactor Aji to element aji. As it is adjoint to A.

Hence adj A= $\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{n1} \\ A_{21} & A_{22} & & A_{n2} \\ \vdots & & & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}$

Example :Let A = $\begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$ find the adjoint A

Solution : The coefficients of A are:

$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} = -18$; $A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$; $A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$;

$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$; $A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$; $A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -2$;

$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -10$; $A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$; $A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$;

Hence adi A = $\begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & 28 \end{bmatrix}$

Example :Let A = $\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 0 \\ 3 & -2 & 1 \end{bmatrix}$ a) find the AdjA ; b) calculate |A| .

Theorem : If A = [a_{11}] Matrix from type (n x n) , then

A .(adj A) = (adj A) A = $|A| I_n$, prove that . (H .W)

Corollary (2) : if $|A| \neq 0$, then $|A^{-1}| = \frac{1}{|A|}$, prove that (H .W)

[3.5] Definition: Be matrix A= [a_{ij}] , upper triangular matrix , if A square matrix which all elements it ,which lies down diagonal equal into zero .

A= $\begin{bmatrix} 2 & -3 & -1 \\ 0 & 5 & 2 \\ o & 0 & 6 \end{bmatrix}$

a matrix A= [a_{ij}] , said lower triangular matrix , if A square matrix which all elements it ,which lies above diagonal equal into zero .

A= $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 5 & 0 \\ 4 & 1 & 6 \end{bmatrix}$

For solution of this Matrix three case

1) - If the number of equations less than the number anonyms .whichever $m < n$ then there are solution is not unique .

2)If the number of equations grand than number anonyms .i, e $m > n$, then have never solution .

3)When m = n , whichever [A] is a square matrix and in this case be unique solution for this system.

Solution Method .

from simplistic of direct methods for solve linear system and summary it by the following steps .

1)we zeroing every elements of first column for matrix [A] except the first element. and that by addition $\frac{-a_{ij}}{a_{11}}$ multiplication in first equation (first row) , into the row (i) all I = 2,3,.., n .

2)we zeroing every elements of second column except the first element and second from the column ,and that by addition $\frac{-a_{i2}}{a_{22}}$ multiplication in second row into the row (i) all 3,4,…, n .

as on continuous with self steps into we get on the matrix [A] by form upper triangular matrix and those we can found every of x value .

Example : Transfer the following Matrix into upper triangular matrix .

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ -2 & 3 & -1 \end{bmatrix}$$

Solution $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ -2 & 3 & -1 \end{bmatrix} \longrightarrow [A \vdots b] = \begin{bmatrix} 2 & 3 & -1\vdots \\ 4 & 4 & -3\vdots \\ -2 & 3 & -1\vdots \end{bmatrix} \begin{matrix} r_1 \\ r_2 \\ r_3 \end{matrix}$

r_1 (-2) $+r_2$ = (2 3 - 1) (- 2) + (4 4 - 3) = (-4 - 6 2) + (4 4 - 3)= (0 -2 -1)

$r_1 + r_3$ = (2 3 -1) (1) + (-2 3 -1) = (0 6 -2)

$[A \vdots b] = \begin{bmatrix} 2 & 3 & -1\vdots \\ 0 & -2 & -1\vdots \\ 0 & 6 & -2\vdots \end{bmatrix} \begin{matrix} r_1 \\ r_2 \\ r_3 \end{matrix}$ r_2 (2) $+r_3$

r_2 (3) $+r_3$ = (0 3 -1) (3) + (0 6 -2) = (0 - 6 -3) + (0 6 - 2) = (0 0 -5)

$[A \vdots b] = \begin{bmatrix} 2 & 3 & -1\vdots \\ 0 & -2 & -1\vdots \\ 0 & 0 & -5\vdots \end{bmatrix} \longrightarrow A = \begin{pmatrix} 2 & 3 & -1 \\ 0 & -2 & -1 \\ 0 & 0 & -5 \end{pmatrix}$

Example (H.W)

Example ; Transfer the following Matrix into upper triangular matrix .

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 3 \\ -2 & 1 & -2 \end{bmatrix}$$